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Supplementary Information for

Optical waveguiding by atomic entanglement in multilevel atom arrays

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Supporting Information Text

Here we present further details on the physics of defect states and we show that subradiant states are robust against (classical) fluctuations in atomic positions.

1. Defect states

In Fig. S1(a) we plot the decay rate of the most subradiant state in the manifold $|F_z| = |F_z^{\max}| - 1$ as a function of atom number N, and classified by the position of the defect atom ($n = 1, 2, 3, 4$). It can be seen that although the position of the defect atom n changes for the most subradiant state overall (minimized over all possible positions n), all of these defect states exhibit qualitatively similar behavior. Considering the most subradiant defect state, we plot in Fig. S1(b) the total population of state $|3\rangle$, which is seen to decrease with atom number as $\sum_j \langle \hat{\sigma}_{33}^j \rangle \sim 1/N^3$. In manifolds with lower angular momentum, defect atoms pile up at the edges of the chain. This is illustrated in Fig. S1(c), where we plot the individual atomic state populations for the most subradiant state of $N = 20$ atoms, in the $|F_z| = |F_z^{\max}| - 2$ manifold. Here, the two defect atoms, characterized by a large population in state $|1\rangle$, are positioned as the two outermost atoms at either of the chain ends.

2. Effect of disorder

In Fig. S2 we plot the decay rate of the most subradiant state in the (a) $|F_z| = |F_z^{\max}|$ and (b) $|F_z| = 0$ manifolds as a function of atom number N for different degrees of spatial disorder. Specifically, here we consider classical disorder, where each atom is randomly displaced around its equilibrium position by a normal distribution of standard deviation σ. We plot the decay rate of the most subradiant eigenstate after performing an average over disorder realizations. The chain lies along z and the disorder is introduced only along this dimension (i.e. there is no disorder in the positions along x, y), as disorder in the perpendicular directions would break the conservation of angular momentum projection, which would make the calculations unmanageable. While increasing disorder leads to an average increase in the decay rate, it is clear from the figure that there are still deeply subradiant states even in the presence of significant fluctuations ($\sigma/d = 0.1$). The number of disorder realizations is 200. For (b), the numerical difficulty of the problem prevents us from going beyond $N = 10 - 12$ for significant disorder. In particular, sparse diagonalization methods seem to fail, which requires one to resort to exact diagonalization instead. Nevertheless, there does not seem to be any drastic difference in how it is affected by disorder, as compared to classical subradiance.

We now briefly discuss the case of quantum disorder. Specifically, we consider the that each atom is tightly trapped in the Lamb-Dicke regime, and in the motional ground state of its trap. When an atom is excited, there is the possibility of an inelastic process, where a photon is emitted and a phonon is created at the same time. As shown in Ref. (1), this leads to an additional decay rate of each atom of $\Gamma' \sim \Gamma_0 \eta^2$, where η is the Lamb-Dicke parameter. This decay process is independent (does not depend on correlations), as the final atomic motional state (with one phonon) is distinguishable.

References

1. PO Guimond, A Grankin, DV Vasilyev, B Vermersch, P Zoller, Subradiant bell states in distant atomic arrays. *Phys. Rev. Lett.* **122**, 093601 (2019).

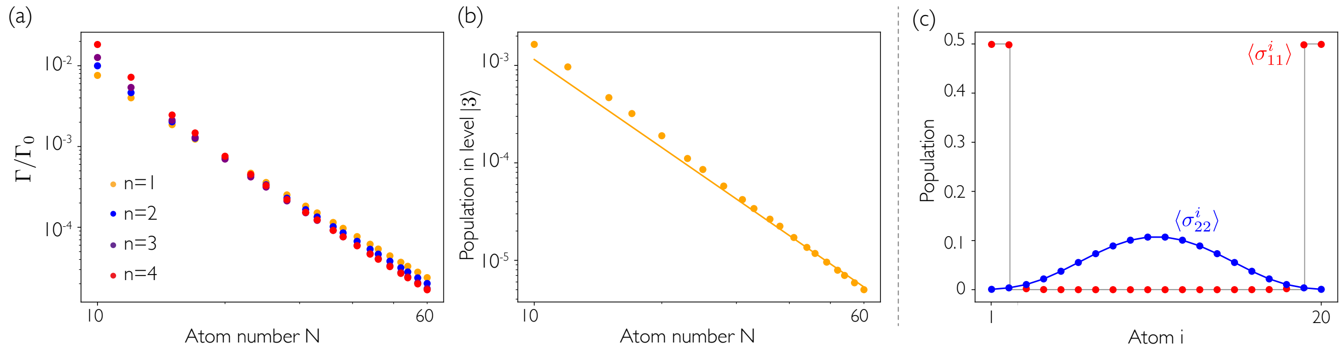

Fig. S1. Details of defect states. **(a)** Scaling of the most subradiant eigenstate with a defect in atom in position n vs. atom number, for $|F_z| = |F_z^{\text{max}}| - 1$. **(b)** Total population in level $|3\rangle$ vs. atom number N, for the most subradiant state in $|F_z| = |F_z^{\text{max}}| - 1$. The continuous line is a guide to the eye showing a scaling of $\sum_j \langle \sigma_{33}^j \rangle \sim 1/N^3$. **(c)** Spatial profile of the populations of levels $|1\rangle$ (red) and $|2\rangle$ (blue) in the most subradiant eigenstate of a chain of $N = 20$ atoms, for $|F_z| = |F_z^{\text{max}}| - 2$. The two defects accumulate at the edge of the chain. The population of $|3\rangle$ is negligible in all atoms. The continuous lines are guides to the eye. For all plots, $d = 0.3\lambda_0$.

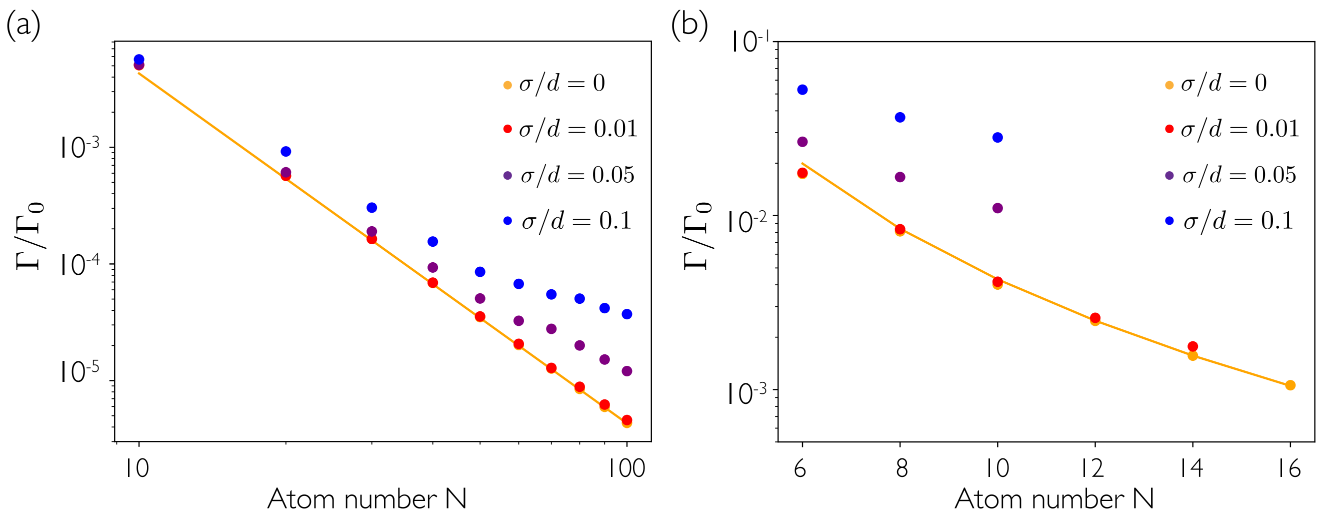

Fig. S2. Influence of classical position disorder σ in the decay rate of the most subradiant state. Averaged scaling of the most subradiant eigenstate vs. atom number for: **(a)** $|F_z| = |F_z^{\max}|$, and **(b)** $|F_z| = 0$ (symmetric states). The continuous lines are guides to the eye showing a scaling of $\Gamma/\Gamma_0 \sim 1/N^3$. The disorder is assumed to be following a normal distribution of standard deviation σ. We consider 200 random configurations for each value of σ. For all plots, $d = 0.3\lambda_0$.